

Rockafellar Convex Analysis PDF

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Rockafellar Convex Analysis: A Deep Dive into the Foundations of Convex Optimization

rockafellar convex analysis represents a cornerstone in the field of optimization and

mathematical analysis, offering profound insights into the behavior of convex functions and their applications. Named after the renowned mathematician R. Tyrrell Rockafellar, this framework has shaped how researchers and practitioners approach complex problems in economics, engineering, machine learning, and beyond. If you've ever wondered why convexity matters so much or how it influences optimization algorithms, exploring Rockafellar's work provides a clear and elegant pathway.

Understanding the Essence of Rockafellar Convex Analysis

At its heart, Rockafellar convex analysis is about studying convex functions, convex sets, and their properties using rigorous mathematical tools. Convexity, simply put, ensures that a line segment connecting any two points on a graph lies above or on the graph itself. This seemingly simple property unlocks powerful guarantees: unique solutions to optimization problems, stability in algorithms, and robustness in modeling.

Rockafellar's contributions go beyond definitions; he provided a comprehensive theory that unifies subdifferentials, conjugate functions, and duality principles into a cohesive whole. His seminal book, "Convex Analysis," first published in 1970, remains an

authoritative reference for scholars diving deep into this subject.

What Makes Rockafellar's Approach Unique?

While convexity was studied before, Rockafellar introduced a systematic and general framework that linked various concepts elegantly. One of his key innovations was formalizing the concept of the subdifferential, a generalization of derivatives for convex functions that may not be differentiable everywhere. This concept is crucial because many real-world problems involve nonsmooth functions, and traditional calculus tools fall short. Moreover, Rockafellar's analysis integrates duality theory with convex functions, enabling the transformation of complex primal optimization problems into often simpler dual problems. This duality framework is invaluable in algorithm design, providing insights into optimality conditions and convergence properties.

Core Concepts in Rockafellar Convex Analysis

To grasp the full power of Rockafellar convex analysis, it's essential to familiarize yourself with several fundamental concepts that underpin the theory.

Convex Sets and Functions

A convex set is a collection of points where, for any two points within the set, the entire line segment joining them also lies within the set. Convex functions, defined on convex sets, satisfy the property:

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$$

]

for all (x, y) in the domain and $\lambda \in [0, 1]$.

This inequality guarantees no "dips" or "valleys" in the graph, making optimization much more tractable.

Subdifferential and Subgradients

Not all convex functions are differentiable. To handle such cases, Rockafellar introduced the subdifferential, denoted $\partial f(x)$, which is the set of all subgradients at a point x . Each subgradient g satisfies:

$$f(y) \geq f(x) + \langle g, y - x \rangle, \quad \forall y$$

This means that the function lies above all its subgradients, generalizing the tangent line concept. Subgradients enable the development of subgradient methods for optimization, crucial in nonsmooth optimization tasks.

Fenchel Conjugates and Duality

Another pillar of Rockafellar convex analysis is the Fenchel conjugate of a convex function f , defined as:

$$f^*(y) = \sup_x \{ \langle y, x \rangle - f(x) \}$$

The conjugate function transforms the problem into a dual space, providing alternative

perspectives and solution techniques. Duality theory stemming from this concept reveals deep connections between primal and dual problems, often simplifying complex constrained optimization scenarios.

Applications of Rockafellar Convex Analysis in Modern Fields

Rockafellar's theoretical framework isn't just abstract mathematics; it underpins many practical applications where optimization plays a central role.

Machine Learning and Data Science

Many machine learning algorithms, from support vector machines to logistic regression, rely on convex loss functions and regularizers that promote sparsity or smoothness.

Rockafellar convex analysis helps in understanding convergence guarantees and designing efficient optimization algorithms like proximal gradient methods and coordinate descent.

In deep learning, although nonconvex optimization dominates, convex analysis tools assist in analyzing layers, initialization, and certain simplified models.

Economics and Game Theory

Convex analysis is fundamental in economic theory for modeling preferences, production sets, and equilibria. Rockafellar's work on duality helps economists analyze cost functions and utility maximization problems, providing robust mathematical underpinnings for equilibrium analysis in markets.

Engineering and Signal Processing

Optimization problems in control theory, network design, and signal recovery often

involve convex constraints and objectives. Rockafellar's framework facilitates the development of algorithms ensuring stability and convergence, essential for real-time and large-scale engineering systems.

Tips for Applying Rockafellar Convex Analysis Effectively

If you're venturing into convex optimization or related fields, integrating Rockafellar's principles can significantly enhance your approach.

Understand the Geometry: Visualizing convex sets and functions helps build

1.
intuition about why certain algorithms work and how constraints shape solutions.

Leverage Subdifferentials: When dealing with nonsmooth problems, use

2.
subgradient methods and understand their convergence properties rooted in Rockafellar's theory.

Explore Duality: Always consider the dual problem; it can offer computational

3.
advantages and deeper insight into sensitivity and robustness.

Utilize Proximal Operators: These operators arise naturally from convex

4.
functions and enable efficient algorithms for large-scale problems.

Challenges and Ongoing Research in Convex Analysis

While Rockafellar convex analysis laid a solid foundation, researchers continue to expand its boundaries. One challenge is extending these theories to infinite-dimensional spaces or stochastic settings, which are common in modern data science and control problems.

Another active area is integrating convex analysis with nonconvex optimization to better understand and solve highly complex real-world problems.

Additionally, computational aspects such as scaling algorithms to massive datasets or designing distributed optimization methods remain vibrant research topics inspired by Rockafellar's foundational work.

Exploring Rockafellar convex analysis offers not just a glimpse into elegant mathematics but a toolkit that empowers solving some of the most pressing optimization challenges today. Whether you're a student, researcher, or practitioner, immersing yourself in these concepts can transform how you approach problems involving convexity and optimization.

Question

Answer

What is Rockafellar's contribution to convex analysis?

Rockafellar is renowned for his foundational work in convex analysis, particularly through his book 'Convex Analysis' which systematically developed the theory of convex functions, subgradients, and duality in

optimization.

What is the significance of

Rockafellar's book 'Convex

Analysis'?

'Convex Analysis' by Rockafellar is considered a seminal

text that rigorously formalized convex analysis as a

mathematical discipline, providing essential tools for

optimization, variational analysis, and economics.

How does Rockafellar's

theory apply to optimization

problems?

Rockafellar's convex analysis framework provides key

concepts such as subdifferentials, conjugate functions,

and duality theory which are fundamental in solving and

analyzing convex optimization problems.

What is the Rockafellar

subdifferential?

The Rockafellar subdifferential generalizes the derivative

for convex functions and is defined as the set of all

subgradients at a point, playing a crucial role in

nonsmooth optimization.

How is Fenchel duality

related to Rockafellar's

work?

Fenchel duality is extensively developed in Rockafellar's

convex analysis framework, providing a powerful tool for

deriving dual problems and understanding the primal-

dual relationship in convex optimization.

Can Rockafellar's convex

analysis be applied to

machine learning?

Yes, Rockafellar's convex analysis concepts underpin

many machine learning algorithms, especially those

involving convex loss functions and regularization

techniques.

What role does convex

conjugation play in

Rockafellar's theory?

Convex conjugation, or the Legendre-Fenchel transform,

is a central concept in Rockafellar's convex analysis,

enabling the study of dual problems and variational

properties of convex functions.

How does Rockafellar's work

influence modern

variational analysis?

Rockafellar's convex analysis provides the mathematical

foundation for variational analysis by extending convex

function theory to study stability, sensitivity, and

generalized differentiation.

What are some applications

of Rockafellar convex

analysis outside pure

mathematics?

Applications include economics for utility and production

theory, engineering for control systems, finance for risk

measures, and computer science for algorithm design.

How can one learn

Rockafellar convex analysis

effectively?

Studying Rockafellar's original book 'Convex Analysis',

supplemented with modern textbooks on convex

optimization and variational analysis, along with practical

problem-solving, is an effective approach.

Rockafellar Convex Analysis: Foundations and Impact on Modern Optimization

rockafellar convex analysis stands as a cornerstone in the realm of mathematical

optimization and variational analysis. Named after the eminent mathematician R. Tyrrell

Rockafellar, this field delves deeply into the properties and applications of convex

functions, sets, and their interactions. Its theoretical framework has profoundly influenced

various disciplines, ranging from economics and engineering to machine learning and

operations research. Understanding Rockafellar's approach to convex analysis is essential

for professionals and researchers aiming to harness the power of convexity in complex

optimization problems.

Understanding Rockafellar Convex Analysis

At its core, Rockafellar convex analysis explores the geometry and calculus of convex

functions and convex sets within real vector spaces. Rockafellar's seminal work,

particularly his 1970 book "Convex Analysis," laid down rigorous foundations that

systematically address convex functions, subdifferentials, conjugate functions, and duality

theory. The methodology emphasizes the interplay between primal and dual problems,

offering tools to analyze and solve optimization tasks with guaranteed convergence and

stability.

Convex analysis in Rockafellar's paradigm is not merely about convexity's geometric

intuition but also involves a robust functional analytic framework. This allows for dealing with infinite-dimensional spaces and nonsmooth functions, broadening the scope beyond classical calculus. The theory is instrumental in characterizing optimality conditions, sensitivity analysis, and algorithmic design, making it indispensable to modern optimization.

Key Concepts in Rockafellar Convex Analysis

Rockafellar's contributions introduced several pivotal concepts that have become standard in convex analysis and optimization literature:

Convex Functions and Sets: Functions whose epigraphs are convex sets and sets

1. sets that contain all line segments between any two points within them.

Subdifferential Calculus: A generalization of gradients to nonsmooth convex

2. functions, enabling the formulation of optimality conditions.

Fenchel Conjugate Functions: A transformation providing a dual representation

3. of convex functions, facilitating duality theory.

Duality Theory: Establishes relationships between primal and dual optimization

4. problems, allowing for alternative problem formulations and solution techniques.

Maximal Monotone Operators: Operators that extend subdifferentials and are

5.

crucial in variational inequalities and partial differential equations.

These elements collectively offer a powerful toolkit for analyzing and solving a wide array of convex optimization problems, especially those involving nonsmooth or infinite-dimensional structures.

Applications and Impact in Optimization and Beyond

The influence of Rockafellar convex analysis extends far beyond pure mathematics. Its theoretical rigor and practical applicability have made it a backbone in optimization, economics, signal processing, and machine learning.

Role in Modern Optimization Algorithms

Many state-of-the-art optimization algorithms draw directly on principles from Rockafellar's convex analysis framework. For example, proximal algorithms, which solve nonsmooth optimization problems by iteratively applying proximal operators, rely heavily on subdifferential calculus and maximal monotone operator theory. These methods are fundamental in large-scale data science, image reconstruction, and sparse signal recovery.

Similarly, duality theory facilitates decomposition methods for large convex optimization problems, enabling distributed computing and parallelization. This is essential in network optimization and large-scale machine learning models where centralized computations are impractical.

Convex Analysis in Economic Theory

Economists utilize Rockafellar's convex analysis to model preferences, production sets, and equilibrium conditions. Convexity assumptions simplify complex economic systems, ensuring the existence and uniqueness of equilibrium points. The subdifferential plays a role in consumer theory, representing marginal utilities in nonsmooth settings, while duality theory relates to cost and production functions.

Comparative Insights: Rockafellar's Approach vs Other Convex Analysis Frameworks

While convex analysis as a discipline has multiple contributors, Rockafellar's approach is distinguished by its comprehensive treatment and generality. Other frameworks might focus exclusively on finite-dimensional settings or smooth functions. In contrast, Rockafellar's work embraces infinite-dimensional spaces and nonsmooth phenomena, broadening applicability.

Moreover, Rockafellar's emphasis on duality and maximal monotone operators provides a unified perspective that integrates optimization, variational inequalities, and partial differential equations. This contrasts with more fragmented treatments that address these topics in isolation.

Strengths and Limitations

Rockafellar convex analysis offers several advantages:

Generality: Applicable to a wide variety of convex problems, including nonsmooth

1.

and infinite-dimensional cases.

Unified Theory: Bridges multiple mathematical disciplines under a cohesive

2.

framework.

Algorithmic Foundations: Underpins modern optimization algorithms and

3.

numerical methods.

However, some challenges exist:

Abstractness: The high level of abstraction can be a barrier to practitioners

1.

without a strong mathematical background.

Computational Complexity: While theoretically robust, some constructs may be

2.

difficult to compute or approximate in practical large-scale problems.

Despite these issues, the framework remains vital for advancing both theory and practice.

Future Directions and Research Trends

As optimization problems grow in complexity and scale, the principles of Rockafellar

convex analysis continue to inspire new research avenues. Extensions into stochastic

convex optimization, robust optimization, and nonconvex generalizations are active areas.

Researchers explore how to relax convexity assumptions while retaining some duality or subdifferential structures, thereby widening applicability.

Additionally, integration with machine learning frameworks, such as deep learning, is a growing focus. Convex analysis provides tools for understanding loss functions, regularization schemes, and convergence properties of algorithms that drive artificial intelligence applications.

The interplay between convex analysis and computational advances, including quantum computing and distributed optimization, also promises to deepen the relevance of Rockafellar's foundational work in coming decades.

Rockafellar convex analysis remains a seminal and evolving field, offering profound insights and tools essential for tackling the optimization challenges of today and tomorrow.

convex functions, convex optimization, subgradients, Fenchel duality, conjugate functions, variational analysis, monotone operators, convex sets, optimization theory, epi-graphs

VARIATIONAL ANALYSIS SERIES GRUNDLEHREN DER MATHE

Variational Analysis Series Grundlehren der Mathe is a comprehensive and authoritative collection that delves into the foundational and advanced aspects of variational analysis within the broader realm of mathematical sciences. As part of the renowned Grundlehren der Mathe series, this collection offers a rigorous exploration of topics essential for researchers, graduate students, and practitioners interested in optimization, functional analysis, and mathematical modeling. In this article, we will explore the significance of the series, its core topics, and how it serves as an invaluable resource for mastering variational analysis.

Understanding Variational Analysis

What is Variational Analysis?

Variational analysis is a branch of mathematics that studies the behavior of functions, sets, and mappings through the lens of optimization and calculus of variations. It encompasses the analysis of problems where the goal is to find extrema (minimum or maximum values) of functionals, often under certain constraints. This field is integral to understanding complex systems where stability, sensitivity, and approximation are crucial.

Historical Development

The development of variational analysis traces back to classical calculus of variations in the 19th century, with significant contributions from mathematicians like Leonhard Euler and Joseph-Louis Lagrange. Over time, the field expanded to include nonsmooth analysis, generalized derivatives, and modern optimization theory, culminating in comprehensive series such as Grundlehren der Mathe.

The Significance of the Series: Grundlehren der Mathe

About the Series

Grundlehren der Mathe (Fundamentals of Mathematics) is a prestigious book series published by Springer, covering a wide array of mathematical disciplines. The series is known for its depth, rigor, and clarity, making complex topics accessible to a broad audience of mathematicians and scientists.

Focus on Variational Analysis Series

The variational analysis series within Grundlehren der Mathe provides an in-depth treatment of core concepts such as subdifferentials, generalized derivatives, variational principles, and stability analysis. It bridges pure theory with practical applications in economics, engineering, and data science.

Core Topics Covered in the Variational Analysis Series

Fundamentals of Variational Analysis

This section introduces the basic concepts, including:

- Set-valued mappings

- Normal cones and tangent cones
- Subdifferentials and generalized gradients
- Variational inequalities

Advanced Variational Principles

Here, the series explores:

- Convex and non-convex analysis
- Stability and sensitivity analysis
- Minimax theorems
- Duality theory

Applications of Variational Analysis

The series highlights applications such as:

- Optimal control theory
- Mathematical economics
- Engineering design optimization
- Data science and machine learning

Why Choose the Variational Analysis Series Grundlehren der Mathe?

Comprehensive and Rigorous Content

The series offers a meticulous presentation of theories, proofs, and methodologies, making it suitable for advanced study and research.

Bridging Theory and Practice

It connects abstract mathematical concepts with real-world applications, providing readers with tools to solve practical problems.

Authoritative Source

Written by leading experts in the field, the series reflects the latest developments and research directions in variational analysis.

How to Utilize the Series for Optimal Learning

Start with Foundational Volumes

Begin with introductory volumes that cover basic concepts to build a solid foundation.

Engage with Examples and Exercises

Practice through exercises and real-world examples provided in the series to deepen understanding.

Integrate with Research and Applications

Apply the theories learned to ongoing research projects or practical problems in science and engineering.

Conclusion

The **variational analysis series grundlehren der mathe** stands as a cornerstone resource for anyone aiming to master the nuances of variational methods and their applications. Its rigorous approach, combined with comprehensive coverage of both fundamental and advanced topics, makes it an essential reference in the field. Whether you are a researcher, academic, or student, engaging with this series will significantly enhance your understanding and ability to apply variational analysis across diverse scientific disciplines.

Further Reading and Resources

- Springer's official page on Grundlehren der Mathe series
- Key textbooks on convex analysis and nonsmooth analysis
- Online courses and seminars on variational methods
- Research journals publishing the latest developments in variational analysis

By exploring the **variational analysis series grundlehren der mathe**, you gain access to a rich repository of knowledge that can propel your understanding of mathematical optimization and its myriad applications.

Variational Analysis Series Grundlehren der Mathematic is a cornerstone reference in the field of mathematical analysis, especially for those delving into the intricate world of variational principles, optimization, and nonsmooth analysis. This comprehensive series, rooted in the prestigious Grundlehren der mathematischen Wissenschaften (Fundamentals of Mathematical Sciences), offers an in-depth exploration of the foundational concepts, advanced theories, and cutting-edge

applications that underpin modern variational analysis. Whether you are a graduate student, a researcher, or a seasoned mathematician, understanding the significance and structure of this series can profoundly enhance your grasp of the subject.

Introduction to Variational Analysis and Its Significance

What is Variational Analysis?

Variational analysis is a broad mathematical discipline that studies the optimization and stability of functionals?functions that map functions, sequences, or more abstract entities to real numbers. It encompasses a variety of tools and concepts used to analyze the behavior of solutions to optimization problems, equilibrium models, and differential equations.

Why is Variational Analysis Important?

- Interdisciplinary Applications: From physics and engineering to economics and machine learning, variational principles underpin many scientific models.
- Theoretical Foundations: It provides rigorous frameworks for understanding stability, sensitivity, and convergence in complex systems.
- Advancement of Optimization Theory: Variational analysis extends classical calculus into nonsmooth and infinite-dimensional settings, enabling the solution of real-world problems with irregular data.

The "Grundlehren der Mathematischen Wissenschaften" Series: An Overview

Historical Context and Prestige

The Grundlehren series, initiated in the early 20th century, is renowned for its authoritative treatment of mathematical subjects. Each volume aims to synthesize existing knowledge while pushing the boundaries of understanding, often authored by leading experts.

Focus on Variational Analysis Series

Within this prestigious series, the Variational Analysis Series stands out as a comprehensive collection that systematically develops the theory from foundational principles to advanced topics.

Structure and Content of the Variational Analysis Series

Core Volumes and Their Focus

The series typically comprises multiple volumes, each dedicated to specific aspects of variational analysis:

- Foundational Principles: Basic concepts, definitions, and classical results.
- Convex Analysis and Monotone Operators: Underpinning theories essential for modern variational methods.
- Nonsmooth Analysis: Techniques for dealing with functions lacking differentiability.
- Optimization and Equilibrium Problems: Variational inequalities, saddle points, and stability analysis.
- Infinite-Dimensional Variational Analysis: Applications in functional analysis and partial differential equations.

Notable Contributors and Their Roles

Leading mathematicians such as R. T. Rockafellar, R. J. Valadier, and others have contributed seminal volumes, each expanding the theoretical landscape and providing rigorous proofs, illustrative examples, and applications.

Key Concepts and Methodologies in the Series

Variational Principles and Theorems

- Ekeland's Variational Principle: A fundamental tool for proving the existence of solutions.
- Convex Duality: Techniques for analyzing primal and dual problems.
- Subdifferentials and Generalized Derivatives: Extending classical derivatives to nonsmooth functions.

Analytical Tools and Techniques

- Monotone Operator Theory: Analyzing operators that preserve order, crucial in solving variational inequalities.
- Clarke's Generalized Gradient: Handling nonsmooth optimization problems.
- Set-Valued Analysis: Studying mappings that assign sets to points, essential for stability analysis.

Applications to Optimization and Control

- Optimality Conditions: Necessary and sufficient conditions in various settings.
- Sensitivity and Stability Analysis: Understanding how solutions change with data perturbations.
- Equilibrium Modeling: Applications in economics, game theory, and network analysis.

How This Series Advances Mathematical Understanding

Bridging Theory and Practice

The series not only develops abstract theory but also demonstrates how to apply these concepts to real-world problems, including variational inequalities in mechanics, image processing, and network flows.

Developing General Frameworks

By synthesizing a wide range of results, the series provides unified frameworks that facilitate the analysis of complex systems beyond classical smooth calculus.

Encouraging Rigorous Mathematical Thinking

Through detailed proofs and comprehensive explanations, the series fosters a deep understanding of the underlying mathematics, essential for innovation and research.

Practical Guidance for Engaging with the Series

Who Should Read It?

- Graduate students starting in variational analysis or optimization.
- Researchers seeking a rigorous reference for theoretical development.
- Practitioners applying mathematical models to engineering, economics, or data science.

Recommended Approach

- Start with Foundational Volumes: Build understanding of core principles and definitions.
- Study Key Theorems and Proofs: Focus on the main results, understanding their assumptions and implications.
- Work Through Examples: Apply theoretical concepts to concrete problems.
- Explore Applications: Connect theory to practical scenarios relevant to your field.
- Participate in Seminars and Discussions: Engage with the mathematical community for deeper insights.

Supplementary Resources

- Textbooks on convex analysis and nonsmooth calculus.
- Research articles citing the series.
- Online courses and lecture notes on variational and convex analysis.

Impact and Future Directions

Continuing Relevance

As mathematical modeling and computational methods evolve, the Variational Analysis Series remains a vital resource for developing new techniques in optimization, machine learning, and data science.

Emerging Topics

Future volumes and research are likely to explore:

- Variational methods in high-dimensional data analysis.
- Machine learning algorithms grounded in variational principles.
- Stochastic variational analysis for uncertain systems.
- Numerical algorithms inspired by variational frameworks.

Final Thoughts

The Variational Analysis Series Grundlehren der Mathematischen Wissenschaften stands as a testament to the depth and richness of modern mathematical analysis. Its rigorous approach, comprehensive coverage, and emphasis on both theory and applications make it an indispensable resource for anyone dedicated to understanding the sophisticated landscape of variational methods. Embracing this series can significantly elevate your mathematical expertise and open new avenues for research and application in diverse scientific domains.

Whether you're exploring foundational concepts or pushing the boundaries of contemporary research, the Variational Analysis Series offers a structured pathway to mastering one of the most dynamic fields in mathematics today.

Question What is the focus of the 'Variational Analysis' series in the Grundlehren der Mathematischen Wissenschaften? The series focuses on the theoretical foundations and advanced topics in variational analysis, including optimization, nonsmooth analysis, and generalized

differentiation. Who are the primary authors or contributors to the 'Variational Analysis' series in Grundlehren der Mathematischen Wissenschaften? Key contributors include R. Tyrrell Rockafellar, Roger J-B Wets, and other leading mathematicians specializing in optimization and variational methods. How does the 'Variational Analysis' series contribute to modern mathematical research and applications? It provides comprehensive theoretical tools that are essential in solving complex optimization problems, economic modeling, control theory, and engineering applications. Is the 'Variational Analysis' series suitable for beginners or mainly for advanced researchers? The series is primarily aimed at graduate students, researchers, and professionals with a strong mathematical background, offering in-depth and advanced coverage of the subject. What are some key topics covered in the 'Variational Analysis' series? Key topics include subdifferential calculus, generalized differentiation, variational inequalities, stability analysis, and duality theory. Are there any prerequisites to understanding the content of the 'Variational Analysis' series? Yes, a solid foundation in real analysis, convex analysis, and linear algebra is recommended to fully grasp the advanced concepts presented. Where can I access or purchase the 'Variational Analysis' series in Grundlehren der Mathematischen Wissenschaften? The series is available through academic publishers such as Springer, and can often be accessed via university libraries or purchased online through their platforms. How does the 'Variational Analysis' series compare to other foundational texts in optimization and analysis? It is considered a comprehensive and rigorous resource that offers a deeper theoretical perspective compared to introductory texts, making it ideal for advanced study and research.

Related keywords: variational analysis, optimization, calculus of variations, functional analysis, mathematical analysis, convex analysis, duality theory, constraint optimization, variational methods, functional spaces

Read the full article online: [Variational analysis series grundlehren der mathe](#)